

Folding Of The Braid Tree With Knot

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Abstract:

In this paper, we will define a new knot called knot braid. Also we will introduce the braid tree with knot together with its adjacent and incidence matrices. The folding of the braid tree with knot will be discussed. Some theorems related to these results are obtained. Also, some applications are introduced.

Keywords: Knot, braid, tree, folding.

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1. Introduction and background:

In mathematics a graph is intuitively a finite set of points in space, called the vertices of the graph, some pairs of vertices being joined by arcs, called the edges of the graph [11].

Tree is a connected graph which contains no cycles or loops is called a tree [11].

In mathematics a knot is a subset of 3- space homeomorphic to the unit circle, while the link is a union of finitely many disjoint knots. The individual knots that make up a link are called its components (so a knot is a link with just one component, i.e. a connected link).

A singular knot is a knot with self-intersection. Fig.(1.1) shows left handed trefoil (with anticlockwise direction), right handed trefoil (with clockwise direction), Hopf link and singular trefoil knot [1,3].



Left hand trefoil Right hand trefoil Hopf link Singular trefoil knot

Fig.(1.1)

We define the tree with knot which is a connected graph that contains number of knots. The field of folding began with S.A. Robertson's work , in 1977 , on isometric folding of Riemannian manifold M into N , which send any piecewise geodesic path in M to a piecewise geodesic path with the same length in N [1,3]. Loop it is edge joining a vertex to itself [3].

If G be a graph without loops, with n - vertices labeled $1, 2, 3, \dots, n$. The "adjacency matrix" $A(G)$ is the $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j [10].

If G be a graph without loops, with n - vertices labeled $1, 2, 3, \dots, n$ and m -edges labeled $1, 2, 3, \dots, m$. The "incidence matrix" $I(G)$ is the $n \times m$ matrix in which the entry in row i and column j is 1 if vertex i is incident with edge j and 0 otherwise[10].

2. Main results:

In 2011, El-Ghoul, M. and Sh. Adel has define a new type of tree called "Tree with knots together" with its adjacent and incidence matrices. Also they discussed some geometric transformations on it; in this paper we will define a new knot called knot braid and the braid tree with knot together. The folding of this tree will be discussed.

2.1. Knot braid:

It is a knot in which its fibers are braids. As shown in the next Fig.(2.1.1).

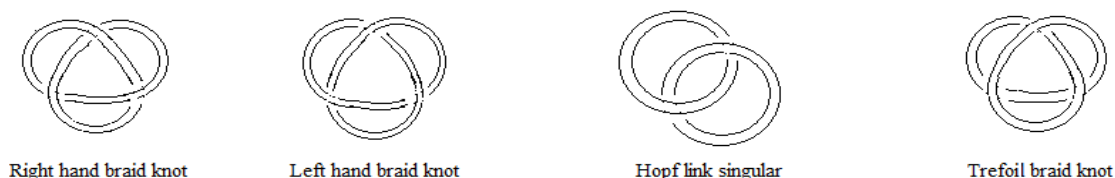


Fig.(2.1.1)

2.2. The braid tree with knot:

It is a tree in which its edges are braid and contains knot braid. See Fig.(2.2.1).

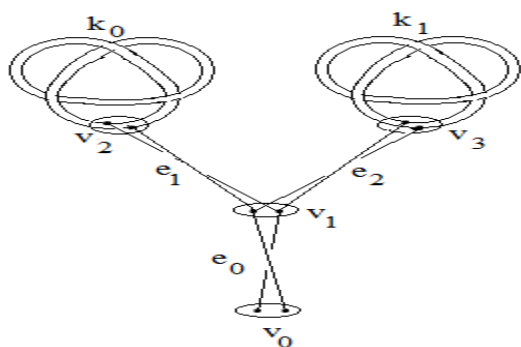


Fig.(2.2.1)

Where its adjacent and incidence matrices are:

$$A(T_{bk}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix},$$

$$I(T_{bk}) = \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix}$$

Where $1^{k_{2f}}$ refers to the existence of 1 knot with 2 fibres and $1^{b_{2f}}$ refers to the existence of an edge which is a braid with 2-fibers.

2.3. Folding to the braid tree with knot:

First: Folding of the knot:

Case (1):

In this case, the folding acts on the length of the knots k_0, k_1 until it reaches the null knot which represent the limits of folding. The result is the braid tree, see Fig.(2.3.1).

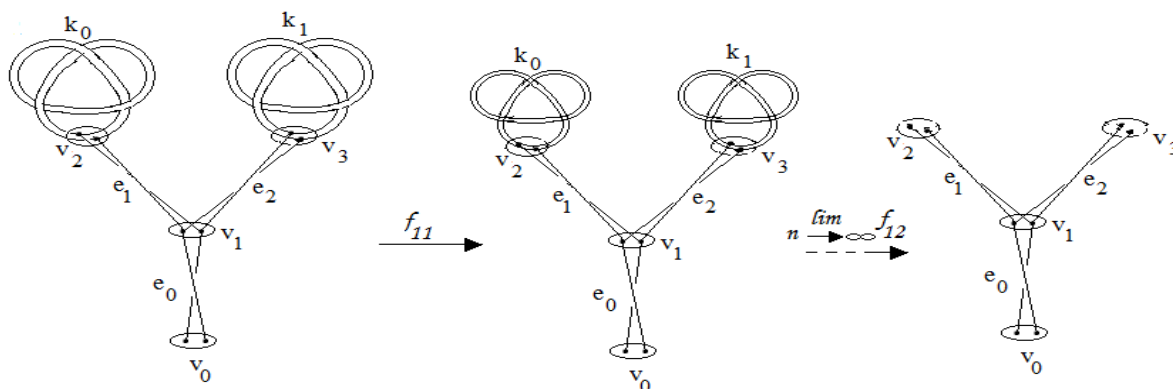


Fig.(2.3.1)

The adjacent matrix will change but the incidence matrix will remain as it is without changing.

$$\begin{aligned}
 A(T_{bk}) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \rightarrow A(f_{11}(T_{bk})) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \dots \dots \\
 &\rightarrow A(\text{lim. } f_{12}(T_{bk})) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \\
 I(T_{bk}) &= \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix} \rightarrow I(f_{11}(T_{bk})) = \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix} \dots \dots \rightarrow I(\text{lim. } f_{12}(T_{bk})) \\
 &= \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix}.
 \end{aligned}$$

Theorem (1)

The folding to the braid tree with knot in case acts on the length of the knot goes to the braid tree.

Proof

The proof is clear from the above discussion, see Fig.(2.3.1).

Case (2):

Here the folding acts on the knots changed it into loops, see Fig.(2.3.2).

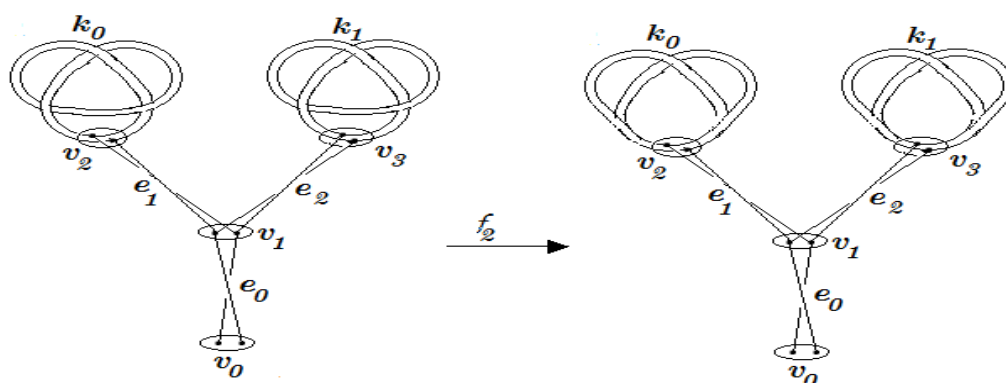


Fig.(2.3.2)

The adjacent matrix only will change as follows:

$$A(T_{bk}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \rightarrow A(f_2(T_{bk})) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 2^{l_{2f}} & 0 \\ 0 & 1 & 0 & 2^{l_{2f}} \end{bmatrix}$$

Such that the index $2^{l_{2f}}$ refers to the existence of two loops which consists of two fibres, and the incidence matrix will remains without changing.

Case (3):

The folding also here acts on the knots k_0, k_1 changing them into loops l_0, l_1 such that $f(k_i) = l_i$, and the braid tree with knots will change to a braid tree with loops, see Fig.(2.3.3).

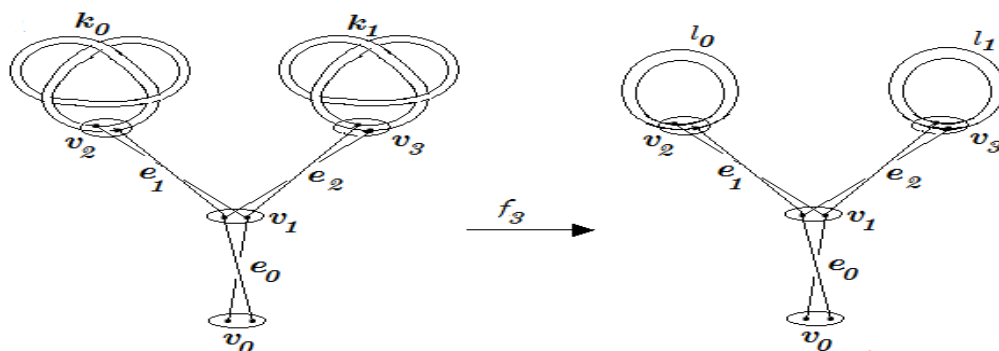


Fig.(2.3.3)

Where the adjacent matrix will be changed as follows:

$$A(T_{bk}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \rightarrow A(f_3(T_{bk})) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{l_{2f}} & 0 \\ 0 & 1 & 0 & 1^{l_{2f}} \end{bmatrix}$$

Where the index $1^{l_{2f}}$ refers to the existence of a loop with 2 fibres, and the incidence matrix will remains without changing.

Second: Folding of the edges:

Case (1):

Here, the folding decreases the length of the edges e_0, e_1, e_2 . The limits of the folding gives a graph without edges, See Fig.(2.3.4).

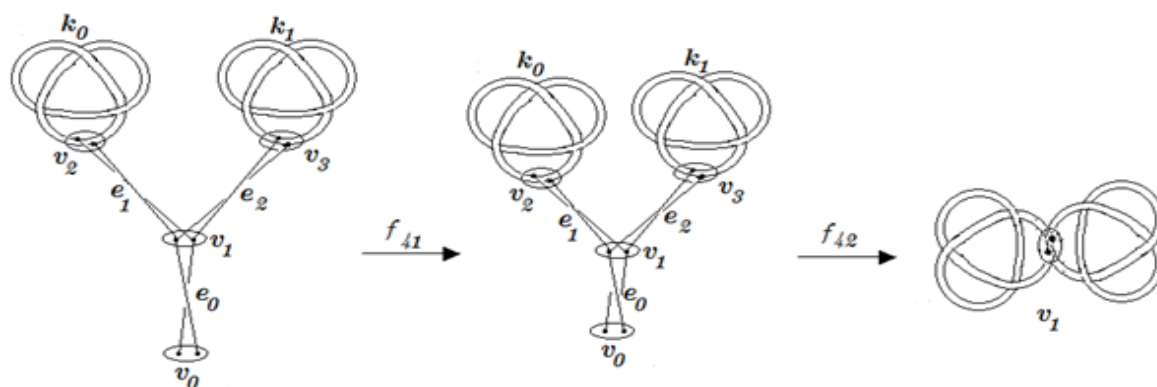


Fig.(2.3.4)

Where the adjacent and incidence matrices will change as follows:

$$A(T_{bk}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \rightarrow A(f_{41}(T_{bk})) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \rightarrow A(f_{42}(T_{bk}))$$

$$= [2^{k_{2f}}]$$

$$I(T_{bk}) = \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix} \rightarrow I(f_{41}(T_{bk})) = \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix} \rightarrow I(f_{42}(T_{bk})) = [0]$$

Theorem (2)

In case the folding decreases the length of the edges the braid tree with knot goes to null knot braid graph.

Proof

The proof is clear from the above discussion, see Fig.(2.3.4).

Case (2):

In this case the folding act on the edge e_0 and change it to a knot k_2 , where $f_5(v_0) = v_1$; $F_5(e_0) = k_2$, see Fig.(2.3.5).

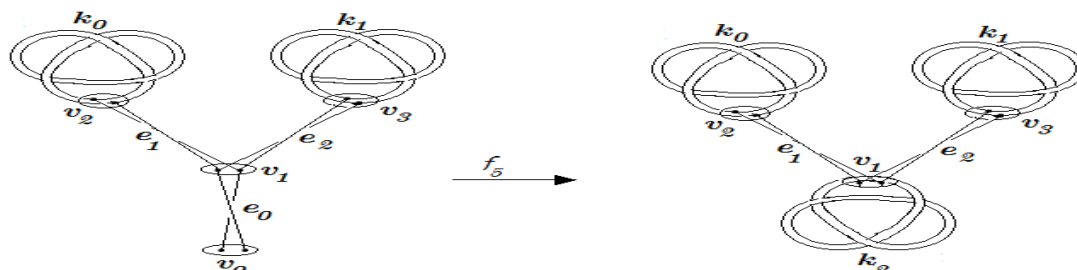


Fig.(2.3.5)

The adjacent and incidence matrices will change as follows:

$$A(T_{bk}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \rightarrow A(f_5(T_{bk})) = \begin{bmatrix} 1^{k_{2f}} & 1 & 1 \\ 1 & 1^{k_{2f}} & 0 \\ 1 & 0 & 1^{k_{2f}} \end{bmatrix}.$$

$$I(T_{bk}) = \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix} \rightarrow I(f_5(T_{bk})) = \begin{bmatrix} 1^{b_{2f}} & 1^{b_{2f}} \\ 1^{b_{2f}} & 0 \\ 0 & 1^{b_{2f}} \end{bmatrix}.$$

Third: Folding of all fibres:

Here, we fold all fibres on each other. The folding gives the tree with knot. See Fig.(2.3.6).

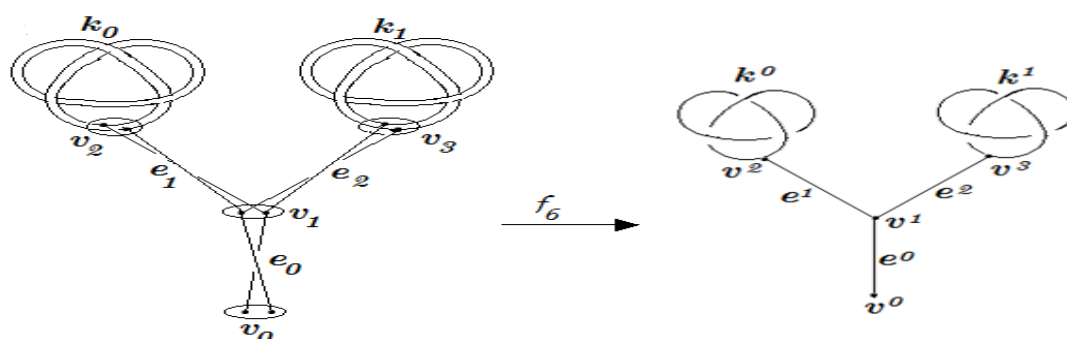


Fig.(2.3.6)

The change in the adjacent and incidence matrices is as follows.

$$A(T_{bk}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^{k_{2f}} & 0 \\ 0 & 1 & 0 & 1^{k_{2f}} \end{bmatrix} \rightarrow A(f_6(T_{bk})) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1^k & 0 \\ 0 & 1 & 0 & 1^k \end{bmatrix}$$

$$I(T_{bk}) = \begin{bmatrix} 1^{b_{2f}} & 0 & 0 \\ 1^{b_{2f}} & 1^{b_{2f}} & 1^{b_{2f}} \\ 0 & 1^{b_{2f}} & 0 \\ 0 & 0 & 1^{b_{2f}} \end{bmatrix} \rightarrow I(f_6(T_{bk})) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Theorem (3)

In case the folding of all fibres the braid tree with knot goes to tree with knot.

Proof

The proof is clear from the above discussion, see Fig.(2.3.6).

3. Application in life:

Some types of natural trees can be considered as braid tree with knot such as Celtic knot tree roots, see the following figures, (3.1) and (3.2).



Fig.(3.1)



Fig.(3.2)

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